Universidade Federal de Rondônia **Departamento de Física**
Cálculo III
2018/2 **Aluno(a):** ____________________
Data: 23/10/2018
Duração da prova: 3h30m Professor: Ariel Adorno de Sousa

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1. (2.0 points) Encontre a natureza das quádricas abaixo, e esboce o seus gráficos:
   (a) $144x^2 + 100y^2 + 81z^2 - 216xz - 540x - 720z = 0$
   (b) $3x^2 + y^2 + z^2 + 4yz + 12x + 2y - 2z + 9 = 0$

2. (1.5 points) Determine a equação do plano passando pelo ponto $P = (3, -1, 2)$, perpendicular à reta determinada por $P_1 = (2, 1, 4) e P_2 = (-3, -1, 7)$. Ache a distância do ponto P ao plano.

3. (1 point) Calcule as derivadas parciais $\partial/\partial x$, $\partial/\partial y$, $\partial/\partial z$ e $\partial/\partial w$ abaixo:
   (a) $f(x, y, z, w) = \dfrac{3x^5 2w^4}{\sqrt{w^2x^3 + y^3z^2}} cotg(xyzw)$
   (b) $f(x, y, z, w) = \dfrac{xy^2z^3w}{1 + x^2 + y^4 + z^6 + w^8}$

4. (2 points) Dê o significado Físico para:
   (a) Divergente
   (b) Gradiente
   (c) Rotacional
   (d) Laplaciano

5. (1.5 points) Cálcule o Gradiente e o laplaciano.
   (a) $f(x, y, z) = \dfrac{y}{\sqrt{x^2 + y^2 + z^2}} + exp\left(-\dfrac{1}{2}(x^2 + y^2 - z^2)\right)$
   (b) $f(x, y, z) = \left(z^2 - \dfrac{x^2 + z^2}{2}\right) cossec(xyz)$

6. (2.0 points) Calcule o divergente e o rotacional das funções abaixo.
   (a) $\vec{f}(x, y, z) = 4xyz\hat{i} - \dfrac{3z^2}{y}\hat{j} + cosx\hat{k}$
   (b) $\vec{f}(\rho, \theta, z) = \dfrac{\rho}{z}\hat{\rho} + sen\theta\hat{\theta} - \rho z\hat{k}$
   (c) $\vec{f}(r, \theta, \phi) = rsen2\theta\hat{r} + rsen^2 2\theta tg\phi\hat{\theta} - \dfrac{\phi}{rcos\theta}\hat{\phi}$

7. (2 points) (questão bônus) Demostre o gradiente em coordenadas cilíndricas

[1]

[1]Um cavalo morto, é um animal sem vida.

Coordenadas cartesianas

Deslocamento infinitesimal $d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}$

Volume infinitesimal $dV = dx\,dy\,dz$

Gradiente $\nabla\Phi = \frac{\partial\Phi}{\partial x}\,\hat{\imath} + \frac{\partial\Phi}{\partial y}\,\hat{\jmath} + \frac{\partial\Phi}{\partial z}\,\hat{k}$

Divergência $\nabla.\,\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

Rotacional $\nabla\wedge\vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$

Laplaciano $\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}$

Coordenadas cilíndricas

Deslocamento infinitesimal $d\vec{r} = dr\,\hat{\boldsymbol{e}}_r + r\,d\phi\,\hat{\boldsymbol{e}}_\phi + dz\,\hat{\boldsymbol{e}}_z$

Volume infinitesimal $dV = r\,dr\,d\phi\,dz$

Gradiente $\nabla\Phi = \frac{\partial\Phi}{\partial r}\,\hat{\boldsymbol{e}}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\phi}\,\hat{\boldsymbol{e}}_\phi + \frac{\partial\Phi}{\partial z}\,\hat{\boldsymbol{e}}_z$

Divergência $\nabla.\,\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$

Rotacional $\nabla\wedge\vec{A} = \left[\frac{1}{r}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{\boldsymbol{e}}_r + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\hat{\boldsymbol{e}}_\phi + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right]\hat{\boldsymbol{e}}_z$

Laplaciano $\nabla^2\Phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$

Coordenadas esféricas

Deslocamento infinitesimal $d\vec{r} = dr\,\hat{\boldsymbol{e}}_r + r\,d\theta\,\hat{\boldsymbol{e}}_\theta + r\,\sin\theta\,d\phi\,\hat{\boldsymbol{e}}_\phi$

Volume infinitesimal $dV = r^2\,\sin\theta\,dr\,d\theta\,d\phi$

Gradiente $\nabla\Phi = \frac{\partial\Phi}{\partial r}\,\hat{\boldsymbol{e}}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\,\hat{\boldsymbol{e}}_\theta + \frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}\,\hat{\boldsymbol{e}}_\phi$

Divergência $\nabla.\,\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$

Rotacional $\nabla\wedge\vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial\phi}\right]\hat{\boldsymbol{e}}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{\boldsymbol{e}}_\theta +$

$+\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\boldsymbol{e}}_\phi$

Laplaciano $\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$