Questão 1)

$$U = e^{y/x} \Rightarrow U = e^{yx^{-1}}$$

$x = 2r\cos(t)$, $y = 4r\sin(t)$

a) $$\frac{du}{dr} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$$

1: $\frac{du}{dx} = -\frac{y e^{y/x}}{x^2}$

2: $\frac{dx}{dr} = 2\cos(t)$

4: $\frac{dy}{dr} = 4\sin(t)$

3: $\frac{du}{dy} = \frac{e^{y/x}}{x}$

Assim substituindo:

$$\frac{du}{dr} = \frac{-y}{x^2} e^{y/x}\, 2\cos(t) + \frac{4e^{y/x}}{x}\sin(t)$$

$$\frac{du}{dr} = \left[-\frac{2y}{x}\cos(t) + 4\sin(t)\right]\frac{e^{y/x}}{x}$$

Assim:

$$\frac{du}{dr} = \left[-\frac{2(4\cancel{r}\sin(t))}{2\cancel{r}\cos t} + 4\sin(t)\right]\frac{e^{\frac{2\cancel{4}r\sin(t)}{2r\cos t}}}{2r\sin(t)}$$

$$\frac{du}{dr} = \left[-4\,tg(t) + 4\sin(t)\right]\frac{e^{2tg(t)}}{2r\sin(t)}$$

$$\frac{du}{dr} = \left[-\frac{\sec(t)}{r} + \frac{2}{r}\right]e^{2tg(t)}$$

$$\frac{du}{dr} = \frac{1}{r}\left[-\sec(t) + 2\right]e^{2tg(t)}$$

fazendo substituição em $t = \pi/3$ $\frac{du}{dr} = 0$

por substituição:

$$U = e^{y/x}$$

$$U = e^{\frac{2\cdot 4r\sin(t)}{2r\cos(t)}} = e^{2tg(t)}$$

$$\boxed{\frac{du}{dr} = 0}$$

$$\frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt} + \frac{du}{dy}\frac{dy}{dt}$$

$\frac{du}{dx} = -\frac{y e^{y/x}}{x^2}$

$\frac{dx}{dt} = -2r\sin(t)$

$\frac{du}{dy} = \frac{e^{y/x}}{x}$

$\frac{dy}{dt} = 4r\cos(t)$

Assim

$$\frac{du}{dt} = \frac{-y}{x^2}e^{y/x}(-2r\sin(t)) + \frac{e^{y/x}}{x}4r\cos(t)$$

$$\frac{du}{dt} = \frac{2yr}{x^2}e^{y/x}\sin(t) + \frac{4r}{x}e^{y/x}\cos(t)$$

substituindo x e y, temos:

$$\frac{du}{dt} = \frac{2(\cancel{8}r\sin(t))}{\cancel{4r^2}\cos^2(t)}\,r\,e^{2tg(t)} + \frac{\cancel{4r}}{\cancel{2r\cos(t)}}e^{2tg(t)}\cancel{\cos(t)}$$

$$\frac{du}{dt} = 2\,tg(t)\cdot\sec(t)\,e^{2tg(t)} + 2e^{2tg(t)}$$

$$\boxed{\frac{du}{dt} = (2\,tg(t)\sec(t) + 2)\,e^{2tg(t)}} = 2\,(tg(t)\sec(t)+1)\,e^{2tg(t)}$$

~~para~~ por substituição:

$$U = e^{2tg(t)}$$

$$\frac{du}{dt} = 2\,(\sec^2(t))\,e^{2tg(t)} \rightarrow =$$

5) $V = \pi x^2 y$; $x = \cos(z)\,\text{sen}(t)$; $y = z^2 e^t$; $\frac{\partial V}{\partial z}$ e $\frac{\partial V}{\partial t}$

1:

$$\frac{\partial V}{\partial z} = \frac{\partial V}{\partial x}\frac{\partial x}{\partial z} + \frac{\partial V}{\partial y}\frac{\partial y}{\partial z}$$

$$\frac{\partial V}{\partial x} = 2\pi x y$$

$$\frac{\partial x}{\partial z} = -\sin(z)\sin(t)$$

$$\frac{\partial V}{\partial y} = \pi x^2$$

$$\frac{\partial y}{\partial z} = 2 z e^t$$

Substituindo:

$$\frac{\partial V}{\partial z} = 2\pi x y(-\sin(z)\sin(t)) + \pi x^2 \cdot 2 z e^t$$

$$\frac{\partial V}{\partial z} = -2\pi x y \sin(z)\sin(t) + 2\pi x^2 z e^t$$

Substituindo x e y. Temos:

$$\frac{\partial V}{\partial z} = -2\pi\left(\cos(z)\sin(t)\right)\cdot\sin(z)\sin(t) + 2\pi\cdot\left(\cos^2(z)\sin^2(t)\right) z e^t$$

$$\frac{\partial V}{\partial z} = -2\pi\cos(z)\sin(z)\sin^2(t) + 2\pi\cos^2(z)\sin^2(t) z e^t$$

$$\frac{\partial V}{\partial z} = -2\pi\sin^2(t)\left(\cos(z)\sin(z) - \cos^2(z)\cdot z e^t\right)$$

Por substituição:

$$V = \pi\left(\cos^2(z)\sin^2(t)\right).z^2.e^t$$

$$\frac{\partial V}{\partial z} = -2\pi e^t z \sin^2(t)\cos z\,(z\sin(z) - \cos(z))$$

Em relação a T

$$\boxed{\frac{\partial V}{\partial T} = \pi e^t z^2 \sin(t)\cos^2(z)\left(\sin(t) + 2\cos(t)\right)}$$

$$\frac{\partial V}{\partial T} = \frac{\partial V}{\partial x}\frac{\partial x}{\partial T} + \frac{\partial V}{\partial y}\frac{\partial y}{\partial T}$$

$\frac{\partial V}{\partial x} = 2\pi x y$; $\frac{\partial x}{\partial T} = \cos z \cos t$

$\frac{\partial V}{\partial y} = \pi x^2$; $\frac{\partial y}{\partial T} = z^2 e^T$

Assim

$$\frac{\partial V}{\partial T} = 2\pi x y \cos z \cos T + \pi x^2 z^2 e^T$$

$$\frac{\partial V}{\partial T} = \pi\left(2 x y \cos z \cos t + x^2 z^2 e^t\right)$$

Substituindo x e y

$$\frac{\partial V}{\partial T} = \pi\left(2\cdot(\cos z\,\text{sen}(t))\cos z\cos t + \cos^2(z)\sin^2(t) z^2 e^t\right)$$

$$\boxed{\frac{\partial V}{\partial T} = \pi e^T z^2 \sin(t)\cos^2(z)(\sin(t) + 2\cos(t))}$$

Questão 3

Questão: 2

a)

b/ o primeiro [illegible]

$$\int_1^2\int_0^1 \frac{y}{1+x^2}\,dx\,dy = \int_1^2 y\,dy \int_0^1 \frac{dx}{1+x^2} = \int_0^1 y\,dy \cdot \left[\text{arctg}\, x\right]_0^1 = \frac{\pi}{4}\,\frac{y^2}{2}\Big|_1^2$$

Assim:

$$I = \frac{\pi}{8}\left(2^2-1^2\right) = \frac{\pi}{8}(4-1) = \cancel{\boxed{\frac{3\pi}{8}}}$$

b)

$$\int_0^e\int_0^y \frac{1}{x^2+y^2}\,dy\,dx = \int_0^e \frac{1}{y}\,\text{arctg}\left(\frac{x}{y}\right)\Big|_0^y dy = \int_0^e \frac{\pi}{4y}\,dy = \frac{\pi}{4}\underbrace{\int_1^e \frac{dy}{y}}_{1} = \frac{\pi}{4}$$

Questão 3

a) $$\int_1^e \int_0^y \int_0^z \frac{z}{x^2+y^2}\, dy\, dx\, dz = \int_1^e \int_0^y z \int_0^z \frac{1}{x^2+y^2}\, dy\, dx\, dz = \int_1^e \int_0^y z \left[\frac{1}{y} \operatorname{arctg}\left(\frac{x}{y}\right)\right]_0^z dy\, dz$$

$$x,\ z,\ y$$

$$J = \int_1^e \int_0^y \frac{z}{y}\left[\operatorname{arctg}\left(\frac{z}{y}\right) - \cancel{\operatorname{arctg}(0)}^{\,0}\right] dy\, dz = \int_1^e \int_0^y \frac{z}{y} \operatorname{arctg}\left(\frac{z}{y}\right) dy\, dz = \int_1^e \frac{1}{y} \underbrace{\int_0^y z \operatorname{arctg}\left(\frac{z}{y}\right) dz}_{\text{Resolvendo por partes}}\, dy$$

$$U = \operatorname{arctg}\left(\frac{z}{y}\right), \quad dv = z\,dz$$

$$dV = \frac{y}{y^2+z^2}\, dz \qquad v = \frac{z^2}{2}$$

Assim, $$J = \frac{z^2 \operatorname{arctg}\left(\frac{z}{y}\right)}{2y} - \frac{1}{y}\int \frac{y z^2}{2(y^2+z^2)}\, dz$$

$$J = \frac{z^2 \operatorname{arctg}(z/y)}{2y} - \frac{1}{2}\int \frac{z^2}{y^2+z^2}\, dz$$

$$J = \frac{z^2 \operatorname{arctg}(z/y)}{2y} + \frac{y^2}{2}\int \frac{1}{y^2+z^2}\, dz - \frac{1}{2}\int dz = z^2 \operatorname{arctg}\left(\frac{z}{y}\right) + \frac{y^2}{2}\int \frac{1}{y^2\left(\frac{z^2}{y^2}+1\right)}\, dz - \frac{1}{2}\int dz$$

Logo $$J = \frac{(y^2+z^2)\operatorname{arctg}\left(\frac{z}{y}\right) - yz}{2y}$$ Substituindo o domínio da integral, temos:

$$J = \int_1^e \frac{1}{y}\left[\frac{(y^2+z^2)\operatorname{arctg}\left(\frac{z}{y}\right) - yz}{2y}\right]_0^y dy = \int_1^e \frac{1}{y}\left[(y^2+y^2)\operatorname{arctg}(1) - y^2 - \cancel{(y^2+0^2)}^{\,0}\cdot 0 - 0\right] dy$$

Continuação 3. ~

$$I = \int_1^e \frac{1}{y}\left(2y^2\frac{\pi}{4} - y^2\right) dy = \int_1^e \left(\frac{2y\pi}{4} - y\right) dy = \int_1^e \frac{y\pi}{2} dy - \int_1^e y\, dy$$

$$I = \frac{\pi}{2}\frac{y^2}{2}\Big|_1^e - \frac{y^2}{2}\Big|_1^e = \left(\frac{\pi}{2} - 1\right)\cdot\frac{y^2}{2}\Big|_1^e = \boxed{\left(\frac{\pi}{4} - \frac{1}{2}\right)\left(e^2 - 1\right)}$$

33 $\displaystyle\int_{-1}^{0}\int_{e}^{2e}\int_{0}^{\pi/3} y\ln(z)\,tg(x)\,dx\,dz\,dy \quad = \int_{-1}^{0} y\,dy\int_{e}^{2e}\ln(z)\,dz\int_{0}^{\pi/3} tg(x)\,dx$

x, z, y

$$J=\int_{-1}^{0} y\,dy\int_{e}^{2e}\ln z\cdot\underbrace{\ln(\sec(x))\Big|_{0}^{\pi/3}}_{\ln(2)}dz = \ln 2\cdot\int_{-1}^{0} y\,dy\int_{e}^{2e}\ln z\,dz = \ln 2\int_{-1}^{0} y\,dy\,\Big[z\ln z - z\Big]_{e}^{2e}$$

$$J=\ln 2\int_{-1}^{0} y\,dy\,\Big[2e\ln 2e - 2e - (\cancel{e\ln e}^{\,0} - e)\Big] = \ln 2\int_{-1}^{0} y\,dy\,\Big[2e\ln 2e - e\Big]$$

$$J=\ln 2\cdot\Big[2e\,\ln 2e - e\Big]\int_{-1}^{0} y\,dy$$

$$J=\ln 2\cdot\left(e\ln 4e^2 - 2e\right)\frac{y^2}{2}\Big|_{-1}^{0}$$

$$=\ln 2\cdot\left(e\ln 4e^2 - 2e\right)\cdot\frac{1}{2}$$

$$=\frac{1}{2}\,e\,\ln(2)\ln(4e^2-2e)\simeq 3\ \boxed{UV}.$$

Questão 4

$$\oint \vec{E}\cdot d\vec{S} = q/\epsilon_0$$

$$\oint \vec{B}\cdot d\vec{S} = 0$$

Usem o teorema do divergente

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 J + \mu_0\epsilon_0 \frac{\partial E}{\partial t}$$

Usem o teorema de Stokes.

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}\vec{E}$$

$$\oint \vec{E}\cdot d\vec{S} = \int (\vec{\nabla}\cdot\vec{E})\,ds = \int \frac{\rho}{\epsilon_0}\,ds$$

= teorema fundamental do cálculo, cálculo

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\oint \vec{B}\cdot d\vec{S} = \int 0\, d\vec{S} = \int (\vec{\nabla}\cdot\vec{B}) \int 0\, ds$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

Usando Stokes.

$$\oint \vec{E}\cdot d\vec{l} = \int (\vec{\nabla}\times\vec{E})\,d\vec{S} = -\iint \frac{\partial B}{\partial t}\,ds \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{B}\cdot d\vec{l} = \int (\vec{\nabla}\times\vec{B})\,ds = \int \left(\mu_0 J + \mu_0\epsilon_0 \frac{\partial}{\partial t}\vec{E}\right) ds \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial}{\partial t}E$$