GABARITO PROVA 1 25/03/2019

PROBLEMA 1

a) $\lim\limits_{x\to -1} \dfrac{x^3+1}{x^2-1} = \lim\limits_{x\to -1} \dfrac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}(x-1)} = \lim\limits_{x\to -1} \dfrac{x^2-x+1}{x-1}$

$= \dfrac{1+(-1)^2+-(-1)}{-1-1} = \boxed{-\dfrac{3}{2}}$

b) $\lim\limits_{x\to 0} \dfrac{tg(x)}{x}$, Assumindo $tg(x) = \dfrac{\sin x}{\cos x}$, temos:

$\lim\limits_{x\to 0} \dfrac{\sin x}{x\cos x} = \underbrace{\lim\limits_{x\to 0} \dfrac{\sin x}{x}}_{\to 1} \cdot \lim\limits_{x\to 0} \dfrac{1}{\cos x} = \lim\limits_{x\to 0} \dfrac{1}{\cos x} = \dfrac{1}{\cos 0}$

$= \dfrac{1}{1} = 1$

c) $\lim\limits_{x\to 0} \dfrac{x^2}{\sin(x)} = \lim\limits_{x\to 0} \dfrac{x\cdot x}{\sin x} = \lim\limits_{x\to 0} \dfrac{x}{\sin x} \cdot \lim\limits_{x\to 0} x$

$= \Big(\underbrace{\lim\limits_{x\to 0} \dfrac{\sin x}{x}}_{\to 1}\Big)^{-1} \cdot \lim\limits_{x\to 0} x = 1^{1} \cdot \lim\limits_{x\to 0} x = 1\cdot 0 = 0$ #

Problema 1

d) $\lim\limits_{x \to 0} \frac{\sin\left(x^2+\frac{1}{x}\right)-\sin\left(\frac{1}{x}\right)}{x}$, feita em sala de aula.

$= 0$

Problema 2, a equação da reta Tg é dada por.

$$y - f(p) = f'(p)(x-p)$$

$$\boxed{y = f'(p)(x-p) + f(p)}.$$

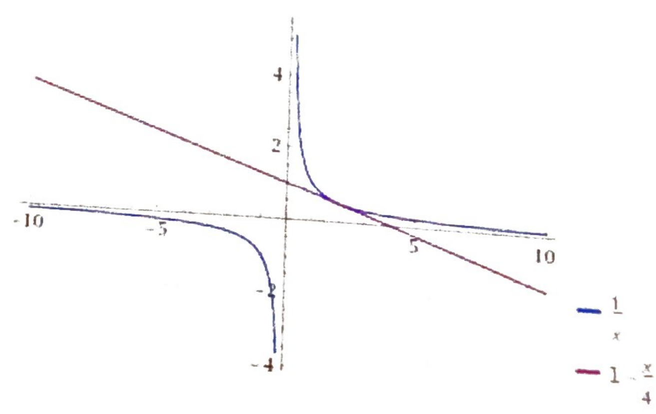
a) $f(x) = \frac{1}{x}$ em $p = 2$

$f'(x) = -\frac{1}{x^2}$, assim:

$f'(p) = -\frac{1}{2^2} = -\frac{1}{4}$

$y = -\frac{1}{4}(x-2) + \frac{1}{2}$

Plot



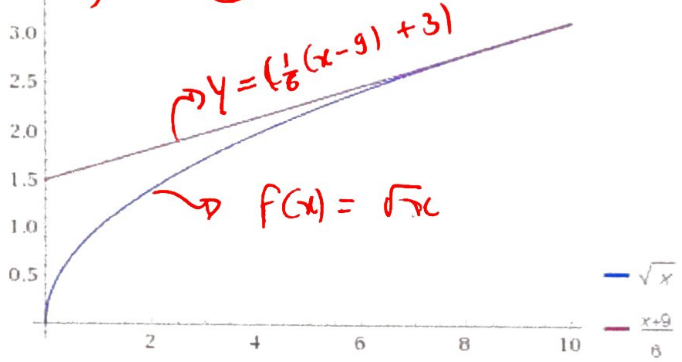
Problema 2

b) $f(x) = \sqrt{x}$, $f'(x) = +\frac{1}{2}\frac{1}{\sqrt{x}}$, $p = 9$

$$f'(p) = \frac{1}{2}\cdot\frac{1}{\sqrt{9}} = \frac{1}{6}$$

Assim:

$$y = \frac{1}{6}(x-9) + 3$$



Problema 3: f', f'', f'''

a) $f(x) = 5x^2 - x^{-3}$

$$\boxed{f'(x) = 10x + 3x^{-4}}$$

$$\boxed{f''(x) = 10 - 12x^{-5}}$$

$$\boxed{f'''(x) = +60x^{-6}}$$

b) $f(x) = \ln(x)$

$$f'(x) = \frac{1}{x}$$

$$f''(x) = -\frac{1}{x^2}$$

$$f'''(x) = +\frac{2}{x^3}$$

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Problema 4

a) $y = (\sin(3x) + \cos(2x))^3$

$y' = 3(\sin(3x) + \cos(2x))^2 \cdot (3 \cdot \cos 3x - 2\sin 2x)$

b) $y = \ln(\sec(x) + tg(x))$

$y' = \dfrac{1}{\sec(x) + tg(x)} \cdot (\sec(x) \cdot tg(x) + \sec^2(x))$

$y' = \dfrac{\sec(x) \cdot \cancel{\{(tg(x) + \sec(x))\}}}{\cancel{\{\sec(x) + tg(x)\}}} = \sec(x)$

c) $g(t) = \dfrac{e^t - e^{-t}}{e^t + e^{-t}} \begin{matrix} f(t) \\ h(t) \end{matrix}$, $g'(t) = \dfrac{f'(t) \cdot h(t) - f(t) \cdot (h'(t))}{h^2(t)}$

$f'(t) = e^t + e^{-t}$ $\quad h^2(t) = (e^t + e^{-t})^2$

$h'(t) = e^t - e^{-t}$

, Assim:

$g'(t) = \dfrac{(e^t + e^{-t}) \cdot (e^t + e^{-t}) - (e^t - e^{-t}) \cdot (e^t - e^{-t})}{(e^t + e^{-t})^2} =$

$g'(t) = \dfrac{(e^t + e^{-t})^2 - (e^t - e^{-t})^2}{(e^t + e^{-t})^2} = \boxed{1 - \dfrac{(e^t - e^{-t})^2}{(e^t + e^{-t})^2}}$

Questão 4

d) $$f(t) = \frac{t e^{2t}}{\ln(3t+1)} \quad \frac{g(t)}{h(t)} \quad , \quad f'(t) = \frac{g' \cdot h - g \cdot h'}{h^2}$$

$$g' = e^{2t} + 2t e^{2t}$$

$$h' = \frac{3}{3t+1} \quad , \quad h^2 = [\ln(3t+1)]^2$$

Assim:

$$f'(t) = \frac{(e^{2t} + 2t e^{2t}) \cdot \ln(3t+1) - \frac{3t e^{2t}}{3t+1}}{[\ln(3t+1)]^2}$$

$$f'(t) = \frac{e^{2t}\left[(1+2t)\ln(3t+1) - \frac{3t}{3t+1}\right]}{[\ln(3t+1)]^2}$$

Questão 5

Feito na sala de aula (última aula antes da prova)

Questão 6

Feito em sala de aula (4ª aula)